
Efficient Beam Tree Recursion (Appendix)

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1 Organization

In Section 2, we describe the settings of all the tasks and datasets that we have tested our models on. In Section 3, we provide additional results on logical inference and sentiment classification. Then, in Section 4, we present an extended survey of related works. In Section 5, we detail our architecture setup including the sequence-interaction models. In Section 6, we provide our hyperparameters. Last, further details on BT-RvNN (if necessary) can be found in the document titled “Beam Tree Recursive Cells” provided in the supplementary (however, we described the salient aspects of the model in the main paper).

2 Task Details

ListOps: ListOps was introduced by Nangia and Bowman [36] and is a task for solving nested lists of mathematical operations. It is a 10-way classification task. Similar to Chowdhury and Caragea [6], we train our models on the original training set with all samples ≥ 100 sequence lengths filtered out. We use the original development set for validation. We test on the following sets: the original test set (near-IID split); the length generalization splits from Havrylov et al. [20] that include samples of much higher lengths; the argument generalization splits from Anonymous [1] that involve an unseen number of maximum arguments for each operator; and the LRA split (which has both higher sequence length and higher argument number) from Tay et al. [52].

Logical Inference: Logical Inference was introduced by Bowman et al. [3] and is a task that involves classifying fine-grained inferential relations between two given sequences in a form similar to that of formal sentences of propositional logic. Similar to Tran et al. [54], our models were trained on splits with logical connectives ≤ 6 . We show the results in OOD test sets with logical connections 10-12. We use the same splits as Shen et al. [44], Tran et al. [54], Chowdhury and Caragea [6].

SST5: SST5 is a fine-grained 5-way sentiment classification task introduced by Socher et al. [50]. We use the original splits.

IMDB: IMDB is a binary sentiment classification task from Maas et al. [31]. We use the same train, validation, and IID test sets as created in Anonymous [1]. We also use the contrast set Gardner et al. [15] and counterfactual set Kaushik et al. [26] as additional test splits.

QQP: QQP¹ [25] is a task of classifying whether two given sequences in a pair are paraphrases of each other or not. As standard Wang et al. [56], we randomly sample 10,000 samples for validation and IID test set such that for each split 5,000 samples are maintained to be paraphrases and the other 5,000 are maintained to be not paraphrases. We also use the adversarial test sets PAWS_{QQP} and PAWS_{WIKI} from Zhang et al. [62].

SNLI: SNLI [2] is a natural language inference (NLI) task. It is a 3-way classification task to classify the inferential relation between two given sequences. We use the same train, development, and IID test set splits as in Chowdhury and Caragea [6]. Any data with a sequence of length ≥ 150 is filtered

¹<https://data.quora.com/First-Quora-Dataset-Release-QuestionPairs>

Table 1: Mean accuracy and standard deviation on the Logical Inference [3] for ≥ 10 number of operations after training on samples with ≤ 6 operations, and on SST5 [50] and IMDB [31]. **Count.** represents counterfactual test split from Kaushik et al. [26] and **Cont.** represents contrast test split from Gardner et al. [15] The best results are shown in bold. Our models were run 3 times on different seeds. Subscript represents standard deviation. As an example, $90_1 = 90 \pm 0.1$

Model	Logical Inference			SST5	IMDB		
	Number of Operations			IID	IID	Cont.	Count.
	10	11	12				
GT-GRC	90.33 ₂₂	88.43 ₁₈	85.70 ₂₄	51.67 _{8.8}	85.11 ₁₀	70.63 ₂₁	81.97 ₅
EGT-GRC	75.79 ₆₁	73.38 ₆₈	69.68 _{7.8}	51.63 ₁₄	86.58 _{2.7}	72 _{9.2}	81.76 ₁₄
CRvNN	94.51 _{2.9}	94.48_{5.6}	92.73 ₁₅	51.75 ₁₁	91.47 _{1.2}	76.98 _{5.8}	83.68 _{7.8}
OM	94.95 ₂	93.9 _{2.2}	93.36 _{6.2}	52.30 _{2.7}	91.69_{0.5}	77.80₁₅	85.38_{3.5}
BT-GRC	95.04 _{2.3}	94.29 _{3.8}	93.36 _{2.4}	52.32_{4.7}	91.29 _{1.2}	75.07 ₂₉	82.86 ₂₃
BT-GRC OS	95.43_{4.5}	94.21 _{6.6}	93.39_{1.5}	51.92 _{7.2}	90.86 _{9.3}	75.68 ₂₁	84.77 ₁₁
EBT-GRC	94.95 _{1.5}	93.87 _{7.4}	93.04 _{6.7}	52.22 ₁	91.47 _{1.2}	76.16 ₁₇	84.29 ₁₂

out from the training set for efficiency. We use also additional test set splits for stress tests. We use the hard test set split from Gururangan et al. [19], the break test set from Glockner et al. [16], and the counterfactual test set from Kaushik et al. [26].

MNLI: MNLI [57] is another NLI dataset, which is similar to SNLI in format. We use the original development sets (match and mismatch) as test sets. We filter out all data with any sequence length ≥ 150 from the training set. Our actual development set is a random sample of 10,000 data-points from the filtered training set. As additional testing sets, we use the development set of Conjunctive NLI (ConjNLI) [41] and a few of the stress sets from Naik et al. [35]. These stress test sets include - Negation Match (NegM), Negation Mismatch (NegMM), Length Match (LenM), and Length Mismatch (LenMM). NegM and NegMM add tautologies containing “not” terms - this can bias the models to classify contradiction as the inferential relation because the training set contains spurious correlations between existence of “not” related terms and the class of contradiction. LenM and LenMM add tautologies to artificially increase the lengths of the samples without changing the inferential relation class.

3 Additional Results

In Table 1, we show that our EBT-GRC model can keep up fairly well with BT-GRC and BT-GRC OS on logical inference [3] and sentiment classification tasks like SST5 [50], and IMDB [15] while being much more computationally efficient as demonstrated in the main paper.

4 Extended Related Works

RvNN History: Recursive Neural Networks (RvNNs) in the more specified sense of building representations through trees and directed acyclic graphs were proposed in [40, 18]. Socher et al. [48, 49, 50] extended the use of RvNNs in Natural Language Processing (NLP) by considering constituency trees and dependency trees. A few works [63, 51, 28, 64] started adapting Long Shot-term Memory Networks [21] as a cell function for recursive processing. Le and Zuidema [29], Maillard et al. [33] proposed a chart-based method for simulating bottom-up Recursive Neural Networks through dynamic programming. Shi et al. [47], Munkhdalai and Yu [34] explored heuristics-based tree-structured RvNNs.

RvNNs can also be simulated by stack-augmented recurrent neural networks (RNNs) to an extent (similar to how pushdown automata can model context-free grammar [42, 27]). There are multiple works on stack-augmented RNNs [4, 60, 32]. Ordered Memory [44] is one of the more modern such examples. More recently, DuSell and Chiang [12, 13] explored non-deterministic stack augmented RNNs and [9] explored other expressive models. Wu [58] presented a survey of latent structure models.

Choi et al. [5] proposed a greedy search strategy based on easy-first algorithm [17, 30] for auto-parsing structures for recursion utilizing STE gumbel softmax for gradient signals. Peng et al. [38] extended the framework with SPIGOT and Havrylov et al. [20] extended it with reinforcement learning (RL). Anonymous [1] extended it with beam search and soft top-k. Chowdhury and Caragea [6], Zhang et al. [61] introduced different forms of soft-recursion.

Top-down Signal: Similar to us, Teng and Zhang [53] explored bidirectional signal propagation (bottom-up and top-down). However, they sent top-down signal in a sequential manner which can be expensive - either it can get slow without parallelization or memory-wise expensive with parallelization of contextualization of nodes in the same height. Our approach in EBT-GAU also has some kinship with BP-Transformer [59]. BP-Transformer allows message passing between a fixed subset of parent nodes and terminal nodes created using a heuristics-based balanced binary tree. Chart-based models can also create sequence contextualized representations [10, 11] but they can be quite expensive by default [1] needing their own separate techniques [22, 23].

Transformers + RvNNs: There have been several approaches to incorporating RvNN-like inductive biases to Transformers. For instance, Universal Transformer [8] introduced weight-sharing and dynamic halt to Transformers. Csordás et al. [7] extended on universal transformer with geometric attention for locality bias and gating. Shen et al. [46] built on weight-shared transformers with high layer depth and group self-attention. Wang et al. [55], Nguyen et al. [37], Shen et al. [45] added hierarchical structural biases to self-attention. Fei et al. [14] biased pre-trained Transformers to have constituent information in intermediate representations. Hu et al. [22] used Transformer as binary recursive cells in chart-based encoders.

5 Architecture details

5.1 Sentence Encoder Models

For the sentence encoder models the architectural framework we use is the same siamese dual-encoder setup as Anonymous [1].

5.2 Sentence Interaction Models

GAU-Block: Our specific implementation of a GAU-block [24] is detailed below. Our GAU-Block can be defined as $\text{GAUBlock}(x, p, G)$. The function arguments are of the following forms: $x \in \mathbb{R}^{n \times d}$, $p \in \mathbb{R}^{l \times d}$ and $G \in \{0, 1\}^{n \times l}$. x accepts the main sequence of vectors that is to serve as attention queries; p accepts either the sequence of intermediate node representations created from our RvNN (for parent attention) or it accepts the same input as x (for usual cases); p serves as keys and values for attention; G accepts either the adjacency matrix in case of parent attention (where $G_{ij} = 1$ iff p_j is a parent of x_i else $G_{ij} = 0$), otherwise, it accepts just the usual attention mask; either way, G serves as an attention mask.

$$x' = \text{LN}(xW_{init} + b_{init}); \quad p' = \text{LN}(pW_{init} + b_{init}) \quad (1)$$

$$u = \text{SiLU}(x'W_u + b_u); \quad v = \text{SiLU}(p'W_v + b_v) \quad (2)$$

$$q = z_q \odot \text{SiLU}(x'W_z + b_z) + zb_q; \quad k = z_k \odot \text{SiLU}(p'W_z + b_z) + zb_k \quad (3)$$

$$A = \text{Softmax}\left(\frac{qk^T + pos}{\sqrt{2d}}, \text{mask} = G\right) \quad (4)$$

$$v' = Av \quad (5)$$

$$o = (u \odot v')W_o + b_o \quad (6)$$

$$g = \text{Sigmoid}([o; x]W_{gate} + b_{gate}) \quad (7)$$

$$\text{out} = g \odot o + (1 - g) \odot x \quad (8)$$

Here, $W_{init} \in \mathbb{R}^{d \times d}$; $W_z \in \mathbb{R}^{d \times d_h}$, $W_u, W_v \in \mathbb{R}^{d \times 2d}$, $b_{init}, b_z, b_o \in \mathbb{R}^d$; $z_q, zb_q, z_k, zb_k \in \mathbb{R}^{d_h}$; $b_u, b_v \in \mathbb{R}^{2d}$, $W_o, W_{gate} \in \mathbb{R}^{2d \times d}$. $[\cdot; \cdot]$ represents concatenation.

LN is layer normalization. pos is calculated using the technique of Shaw et al. [43] using relative tree height distance for parent attention, or relative positional distance for usual cases.

114 **GAU Sequence Interaction Setup:** Let GAUStack represent some arbitrary number of compositions
 115 of GAUBlocks (multilayered GAU block). GAUStack has the same function arguments as GAUBlock.
 116 Given two sequences (x_1, x_2) and their corresponding attention masks (M_1, M_2) as inputs where
 117 $x_1 \in \mathbb{R}^{n_1 \times d}$, $x_2 \in \mathbb{R}^{n_2 \times d}$, $M_1 \in \{0, 1\}^{n_1 \times n_1}$, $M_2 \in \{0, 1\}^{n_2 \times n_2}$, the GAU setup can be expressed
 118 as:

$$inp = [CLS + seg_1; x_1 + seg_1; SEP; CLS + seg_2, x_2 + seg_2] \quad (9)$$

$$r = \text{GAUStack}(x = inp, p = inp, G = [M_1; M_2]) \quad (10)$$

$$\alpha = \text{Softmax}(\text{GELU}(rW_1 + b_1)W_2 + b_2) \quad (11)$$

$$cls' = \sum_i \alpha_i r \quad (12)$$

$$logits = \text{GELU}(cls'W_1^{logits} + b_1^{logits})W_2^{logits} + b_2^{logits} \quad (13)$$

123 Here, $CLS, SEP, seg_1, seg_2 \in \mathbb{R}^{1 \times d}$ are randomly initialized trainable vectors; seg_1, seg_2 are
 124 segment embeddings. $W_1 \in \mathbb{R}^{d \times d}$, $W_2 \in \mathbb{R}^{d \times 1}$; $b_1, b_2, b_1^{logits} \in \mathbb{R}^d$; $b_2^{logits} \in \mathbb{R}^c$; $W_1^{logits} \in$
 125 $\mathbb{R}^{d \times d}$, $W_2^{logits} \in \mathbb{R}^{d \times c}$. c is the number of classes for the task.

126 **EGT-GAU Sequence Interaction Setup:** EGT-GAU starts from the same input as above. Let us
 127 also assume we have the EGT-GRC(x) module which takes a sequence of vectors $x \in \mathbb{R}^{n \times d}$ as
 128 the input to recursively process and outputs (cls, p, G) where $cls \in \mathbb{R}^{1 \times d}$ is the root representation,
 129 $p \in \mathbb{R}^{l \times d}$ is the sequence of non-terminal representations from the tree, and $G \in \{0, 1\}^{n \times l}$ is the
 130 adjacency matrix for parent attention (i.e., $G_{ij} = 1$ iff p_j is a parent of x_i , else $G_{ij} = 0$). Technically,
 131 tree height information is also extracted for relative position but we do not express that explicitly for
 132 the sake of brevity. With these elements, EGT-GAU can be expressed as below:

$$cls_1, p_1, G_1 = \text{EGT-GRC}(x = x_1); \quad cls_2, p_2, G_2 = \text{EGT-GRC}(x = x_2) \quad (14)$$

$$x'_1 = \text{GAUStack}_1(x = x_1, p = p_1, G = G_1); \quad x'_2 = \text{GAUStack}_1(x = x_2, p = p_2, G = G_2) \quad (15)$$

$$cls'_1 = \text{GELU}(cls_1W_1^{cls} + b_1^{cls})W_2^{cls} + b_2^{cls}; \quad cls'_2 = \text{GELU}(cls_2W_1^{cls} + b_1^{cls})W_2^{cls} + b_2^{cls} \quad (16)$$

$$inp = [cls'_1 + seg_1; x'_1 + seg_1; SEP; cls'_2 + seg_2, x'_2 + seg_2] \quad (17)$$

$$r = \text{GAUStack}_2(x = inp, p = inp, G = [M_1; M_2]) \quad (18)$$

137 Everything else after eqn. 18 is the same as eqn. 11 to 13. $SEP, seg_1, seg_2 \in \mathbb{R}^{1 \times d}$; seg_1, seg_2 are
 138 segment embeddings as before. $W_1^{cls}, W_2^{cls} \in \mathbb{R}^{d \times d}$; $b_1^{cls}, b_2^{cls} \in \mathbb{R}^d$.

139 **EBT-GAU Sequence Interaction Setup:** This setup is similar to that of EGT-GAU but with a few
 140 changes. EBT-GAU uses EBT-GRC as a module instead of EGT-GRC. EBT-GAU returns outputs of
 141 the form (cls, bp, bG, s) where $cls \in \mathbb{R}^{1 \times d}$ is the beam-score-weighted-averaged root representation,
 142 $bp \in \mathbb{R}^{b \times l \times d}$ are the beams (beam size b) of sequences of non-terminal representations from the
 143 tree, $bG \in \{0, 1\}^{b \times n \times l}$ are the beams of adjacency matrices for parent attention, and $s \in \mathbb{R}^b$ are the
 144 softmax-normalized beam scores. Let NGAUStack represent the same function as GAUStack but
 145 formalized for batched processing of multiple beams of sequences. With these elements, EBT-GAU
 146 can be expressed as:

$$cls_1, bp_1, bG_1, s_1 = \text{EBT-GRC}(x = x_1); \quad cls_2, bp_2, bG_2, s_2 = \text{EBT-GRC}(x = x_2) \quad (19)$$

$$bx_1 = \text{repeat}(x_1, b); \quad bx_2 = \text{repeat}(x_2, b) \quad (20)$$

$$bx'_1 = \text{NGAUStack}_1(bx_1, bp_1, bG_1); \quad bx'_2 = \text{NGAUStack}_1(bx_2, bp_2, bG_2) \quad (21)$$

$$x'_1 = \sum_i s[i] \cdot bx'_1[i]; \quad x'_2 = \sum_i s[i] \cdot bx'_2[i] \quad (22)$$

150 Everything else after eqn. 22 is the same as the equations 16-18 followed by the equations 11 to 13.
 151 $\text{repeat}(x, b)$ changes $x \in \mathbb{R}^{n \times d}$ to $bx \in \mathbb{R}^{b \times n \times d}$ by batching the same x for b times.

6 Hyperparameter details

For sentence encoder models, we use the same hyperparameters as [1] (the preprint of the paper is available in the supplementary in anonymized form) for all the datasets. The only new hyperparameter for EBT-GRC is d_s which we set as 64; otherwise the hyperparameters are the same as that of BT-GRC or BT-GRC OS. We discuss the hyperparameters of the sequence interaction models next. For EBT-GAU/EGT-GAU, we used a two-layered weight-shared GAU-Blocks for NGAUStack₁/GAUStack₁ and a three-layered weight-shared GAU-Blocks for GAUStack₂ (for parameter efficiency and regularization). GAU uses a five-layered GAU-Blocks (weights unshared) for GAUStack so that the parameters are similar to that of EBT-GAU or EGT-GAU. We use a dropout of 0.1 after the multiplication with W_o in each GAUBlock layer and a head size d_h of 128 (similar to Hua et al. [24]). For relative position, we set $k = 5$ (k here corresponds the receptive field for relative attention in Shaw et al. [43]) for normal GAUBlocks and $k = 10$ for parent attention (since parent attention is only applied to higher heights, we do not need to initialize weights for negative relative distances). Other hyperparameters are kept same as the sentence encoder models. The hyperparameters of MNLI, SNLI, and QQP are shared. Note that all the natural language tasks are trained with fixed 840B Glove Embeddings [39] as in Anonymous [1]. All models were trained in a single Nvidia RTX A6000. The code is available in the supplementary.

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